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USINGANALYTICALANDNUMERICALAPPROACHESFORCRACKMONITORINGBASEDONTHEFUNDAMENTALFREQUENCYOFCRACKEDBEAMS

ABSTRACT

Dynamic behavior of beams, in elastic range, is affected by cracks which have been created during the process of manufacturing or through previous loadings. Cracks in a structure reduce its natural frequencies because it becomes more flexible. So, analysis and detecting of mechanical defects by monitoring of vibration behavior can be a good way for detecting cracks with respect to their size and location. For this purpose, in the current research, the first natural frequency (fundamental frequency) of Euler–Bernoulli beam, in bending vibration, is obtained in two ways, analytical and numerical approaches. In analytical method, flexible influence function was used. The influence of the crack was represented by an elastic rotational spring connecting the two segments of the beam at the cracked section. ANSYS workbench has been used regarding the numerical solution. The results of ANSYS workbench have been validated through comparison of them with the results of analytical solution. There is a good agreement between the analytical and numerical approaches. After validating, a few applied examples were simulated by FEM, regarding the location and size of the crack and the resulting fundamental frequency. The obtained results provide an insight into how fundamental frequency of the cracked structures will help in crack detection and monitoring. The obtained results are useful in the field of Diagnostic, Prognostic and Health Management (DPHM).

Keywords Fundamental Frequency, Euler-Bernoulli Beam, Bending Vibration, Crack Monitoring.

1. INTRODUCTION

Aerospace structures are the most important part of engineering structures, so their analysis and design are very important. In the analysis of aerospace structures, in addition to the inspections of strength of materials, a fracture mechanic analysis is required too. In structures, there are many different ways in which the cracks form and grow. Cracks can appear in structural elements as a consequence of initial defects within the material, depending on the chosen technique of manufacturing and processing, or caused by fatigue during their operational life. In this relation, fatigue crack growth is one of the most important factors. Cracks in a structure reduce its natural frequencies because it becomes more flexible, so, the measurement of these frequencies by monitoring of the vibration behavior of the structural component can be a good way for detecting cracks with respect to their presence, size and location[1-7]. In addition, it is possible to detect the time of required maintenance or the replacement of structural component. Modal shapes and natural frequencies can be obtained by adopting experimental or theoretical methods.

In this paper monitoring of cracks based on fundamental frequency of cracked beams has been studied by authors. This approach has been followed by obtaining damage indices based on the difference in the fundamental frequencies. These damage indices are then correlated to the size and location of cracks in the beam under bending vibration load. For this purpose, in the current research, the first natural frequency (fundamental frequency) of Euler–Bernoulli beam, in bending vibration, is obtained in two ways, analytical and numerical approaches. In analytical method, flexible influence function was used. The influence of the crack was represented by an elastic rotational spring connecting the two segments of the beam at the cracked section. ANSYS workbench has been used regarding the numerical solution. The results of ANSYS workbench have been validated through comparison of them with the results of analytical solution. There is a good agreement between the analytical and numerical approaches. After validating, a few applied examples were simulated by FEM, regarding the location and size of the crack. The obtained results are useful in the field of Diagnostic Prognostic and Health Management (DPHM)[8-14]. Vibration-based damage detection and vibration-based condition monitoring with regard to the beams and other applications are subjects that have been pointed by many authors[15-23].

2. FORMULAS AND METHODS

2-1. Equations and Mathematical Formulas

Referring to Meirovitch,[24] the vertical displacement, $y(x,t)$, at any point, x, of an Euler-Bernoulli beam of length L, subjected to an external load $P(x,t)$, as shown in Figure 1, is expressed as follows:

$$y(x,t) = \int_0^L c(x,\xi)P(\xi,t)d\,\xi \quad (1)$$

Where $c(x,\xi)$ is the flexibility influence function defined as the vertical displacement of the considered point,x, due to a unit load applied at the point of abscissa ξ. At any point of$x$, the beam has a cross-sectional area$A(x)$, an area moment of inertia about the neutral axis, $I(x)$, and a mass per unit length, $m(x)$ ($m(x) = \rho(x)A(x)$, $\rho(x)$being the mass density of the beam material).
For a beam undergoing free vibration, the load represents the inertia force, so that

$$P(\xi,t) = -m(\xi)\frac{\partial^2 y}{\partial t^2} \quad (2)$$

Where $m(\xi)$ is the mass per unit length. Assuming a harmonic motion of frequency ω during free vibration,

$$y(x,t) = u(x)sin\,(\omega t) \quad (3)$$

$u(x)$being the transverse deflection of the beam, equation (1) reduces to

$$u(x) = \omega^2 \int_0^L c(x,\xi)m(\xi)u(\xi)d\,\xi \quad (4)$$

Using new functions, as follows:

$$\lambda = \omega^2, K_1(x,\xi) = c(x,\xi)\sqrt{m(x)}\sqrt{m(\xi)}, (x) = u(x)\sqrt{m(x)}\,, \quad (5)$$

Equation (4) becomes:

$$r(x) = \lambda \int_0^L K_1(x,\xi)r(\xi)d\,\xi \quad (6)$$

The last equation is a homogeneous Fredholm integral equation with symmetric kernel,$K_1(x,\xi)$, represents a classical eigenvalue problem formulated in integral form: solutions $r_n(x)$ (eigenfunctions) exist only for a discrete set of values λ_n(eigenvalues)[25]. Penny and Reed[26] developed a procedure to obtain the upper and lower bounds of the first eigenvalue,λ_1.

$$(\frac{1}{J_n})^{\frac{1}{n}} < \lambda_1 < \frac{J_{n-1}}{J_n} \quad (7)$$

With

$$J_n = \int_0^L K_n(x,x)\,dx \quad (8)$$

$$K_n(x,\xi) = \int_0^L K_{n-1}(x,\eta)\,K_1(\eta,\xi)d\eta \quad (9)$$

Since $\lambda_1 = \omega_1^2$, for the cases n=1 and n=2, the first lower bound of the actual value of the fundamental frequency, ω_1, can be deduced as $(1/J_1)^{1/2}$ and $(1/J_2)^{1/4}$, respectively. It is clear that the larger value of n can provide a closer approximation for the bounds. For the case of uniform simply supported and fixed-fixed beams, as well as for cantilever beams of varying width and thickness, Penny and Reed[26] found that the second lower bound is a sufficiently good approximation to the true value of λ_1.

2-2. Analytical Model of Cracked Beam

In general, the vibrational behavior of the cracked Euler-Bernoulli beam is non-linear, due to the opening and closing of the crack. Figure 2(a) shows auniform beam with a transverse edge crack. The depth of crack is *a* and has been placed at a distance *b* from the left support. The length and width of the beam are *L* and *W* respectively. The cracked beam has been modeled as two beams connected by a rotational elastic spring at the crack section[27], as shown in Figure2(b). The subscripts 1 and 2 refer to the left and right segment of rotational spring respectively. In this study regarding the calculation of the fundamental frequency of the beam for bending vibration it is assumed that the crack remains always open, similar to the work of Chati*et al*[28].

If the transverse deflection of the beam and its corresponding slope are defined by *u(x)* and *θ(x)*, the used model introduces a discontinuity in the slope of the beam at the crack section which is proportional to the bending moment transmitted through it $M_f(b)$[29]:

$$u_2'(b) - u_1'(b) = \Delta\theta = C_m M_f(b) \qquad (10)$$

Where$(.)'$denotes derivation by x and$C_m$is the flexibility constant of the spring.$C_m$can be calculated by

$$C_m = \frac{W}{EI}\Phi(\alpha, geometry\ of\ the\ cross-section) \qquad (11)$$

E being Young's modulus of the beam material,$I$is the moment of inertia of the normal section and$\alpha = \frac{a}{W}$is the crack ratio.Φ is a function that (according to the theory of Fracture Mechanics and, in the case of a rectangular section) takes the form[25]:

$$\Phi(\alpha) = 2\left(\frac{\alpha}{1-\alpha}\right)^2(5.93 - 19.69\alpha + 37.14\alpha^2 - 35.84\alpha^3 + 13.12\alpha^4) \qquad (12)$$

At crack place (x=b) continuous displacement, bending moment and shear force are other kinematic conditions that should be satisfied. These conditions are defined as:

$$u_1(b) = u_2(b), u_1''(b) = u_2''(b), u_1'''(b) = u_2'''(b) \qquad (13)$$

In this paper the regarded support conditions are as follows:

For x=0: $u_1 = 0$, $\frac{du_1}{dx} = 0$ (14)

For x=L: $\frac{d^2u_2}{dx^2} = 0$, $\frac{d^3u_2}{dx^3} = 0$ (15)

The vertical displacement *c(x,ξ)* and the slope of the transverse deflection of the beam, *θ(x,ξ)*, by taking account of both the boundary and the kinematic conditions can be calculated by

For$x$≤b

$$\theta(x,\xi) = \theta(0) + \int_0^x \frac{M_f(x,\ \xi)}{EI}dx \qquad (16)$$

$$c(x,\xi) = \theta(0)x + \int_0^x \frac{M_f(x,\ \xi)}{EI}xdx \qquad (17)$$

For$x$>b

$$\theta(x,\xi) = \theta(0) + \int_0^x \frac{M_f(x,\ \xi)}{EI}dx + \Delta\theta \qquad (18)$$

$$c(x,\xi) = \theta(0)x + \int_0^x \frac{M_f(x,\ \xi)}{EI}xdx + \Delta\theta(x-b) \qquad (19)$$

Where*θ(0)* is the slope of the beam at the left support, *M_f(x, ξ)* the bending moment at the point*x*produced by a unit load applied at point*ξ*and*Δθ*the discontinuity of the slope at the cracked section, which is proportional to the bending moment transmitted by this section. By using the following dimensionless variables:

$$\bar{W} = \frac{W}{L} \ , \ \bar{b} = \frac{b}{L} \ , \ \bar{\xi} = \frac{\xi}{L} \ , \ \bar{x} = \frac{x}{L} \qquad (20)$$

The general relation for the function *c(x, ξ)* can be written as

$$c(x,\ \xi) = \frac{L^3}{EI}\bar{c}_i(\bar{x},\bar{\xi};\bar{b},\bar{W},\alpha) \qquad (21)$$

According to the required calculations[26], approximate values (lower bounds) of the fundamental frequency can be obtained as

First lower bound of fundamental frequency

$$\omega_1 > \left(\frac{1}{J_1}\right)^{\frac{1}{2}} = f_1(\bar{b},\bar{W},\alpha)\sqrt{\frac{EI}{mL^4}} \qquad (22)$$

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Second lower bound of fundamental frequency:

$$\omega_1 > \left(\frac{1}{J_2}\right)^{\frac{1}{4}} = f_2(\bar{b}, \bar{W}, \alpha)\sqrt{\frac{EI}{mL^4}} \tag{23}$$

The functions f_1 and f_2 have different values, corresponding to the type of boundary conditions[26]. In this study, in relation to the considered beam (Figure 3), the functions f_1 and f_2 have the following expressions.

$$f_1 = 3.4641\left(\frac{1}{1+4\bar{W}(1-\bar{b})^3\Phi}\right)^{1/2} \tag{24}$$

$$f_2 = 3.5154\left(\frac{1}{1+\frac{8}{33}\bar{W}(1-\bar{b})^4(33+41\bar{b}+29\bar{b}^2+2\bar{b}^3+70\bar{W}(1-\bar{b})^2\Phi)\Phi}\right)^{1/4} \tag{25}$$

3. EXPERIMENTAL

Numerical Simulation

Figure 3 shows a model of the cracked cantilever beam that was used for evaluating the fundamental frequency of the beam. For this purpose a rectangular beam of width B=10mm, length L=200mm, and thickness W=10mm is studied. Material properties of the beam are as follows:

mass density: ρ=7850 kg/m^3,Young's modulus: E=200GPa,Poisson's ratio: v= 0.3

As shown in Figure 3, the parameters b, a and c indicate the position and dimensions of the crack, respectively. These parameters, in their dimensionless forms, are defined as b/L, a/W and c/B. In this study, ω_1 and ω_1* were selected as the fundamental frequency of the cracked and normal beam, respectively. With regard to the numerical solution, ANSYS workbench was used. Figure 4 shows the variation of fundamental frequency with crack ratio ($\frac{a}{W}$) of the cantilever beam. The other dimensionless parameters have a constant value, $\frac{c}{B} = 1$, $\frac{b}{L} = 0.75$ (c=10 mm, b=150 mm). To verify and validate the obtained results, the solution produced by the ANSYS Workbench was compared to the analytical solutions of the model, according to equation (22) and (23). The results show a good agreement between the numerical and analytical solutions. Numerical simulation shows that the results of equation (23) are more accurate than obtained by equation (22). Figure 5 shows the Variation of fundamental frequency with different $\frac{a}{W}$ and $\frac{b}{L}$ values of the cantilever beam. The other dimensionless parameter has a constant value, $\frac{c}{B} = 1$ (c=B=10 mm). Figure 4 is an especial case of Figure 5, in which,$\frac{b}{L} = 0.75$. Figure 6 shows the variation of fundamental frequency with different $\frac{a}{W}$ and $\frac{c}{B}$ values of a Cantilever beam. The other dimensionless parameter has a constant value, $\frac{b}{L} = 0.75$ (b=150 mm). Figure 4 is an especial case of Figure 6, in which, $\frac{c}{B} = 1$. The data structure of 3D surfaces, as shown in Figure 5 and Figure 6, indicates the fundamental frequency of the cracked cantilever beam depending on the position and dimensions of the crack (a, b, c). The obtained results provide an insight into how fundamental frequency of the cracked structures will help in crack detection and monitoring. These types of data are applicable in relation to the area of Diagnostics Prognostics and Health Management (DPHM).

4. RESULTS AND DISCUSSION

According to obtained results from analytical and numerical solutions,which are shown in Figure 4 while $\frac{c}{B} = 1$, $\frac{b}{L} = 0.75$, by using equation (23), the software results are validated. So the model which is made in Ansys workbench is trusted.

For $\frac{c}{B} = 1$and variation in $\frac{a}{W}$ and$\frac{b}{L}$ values, the frequencies ratios are recorded and shown in Figure 5. By increasing in $\frac{a}{W}$ and fixed $\frac{b}{L}$ the ratio between frequency in cracked beam and frequency in normal beam is decreased. This means that crack growing in this direction cause lower stiffness and decrease the natural frequency of the beam. In addition, it is shown that by increasing in $\frac{b}{L}$ the intensity of decreasing in frequencies ratios became lower that shows how stiffness matrix changes and how it influence the main beam.

For $\frac{b}{L} = 0.75$ the results of frequencies ratios between the cracked beam and normal beam for the variations of $\frac{a}{W}$ and $\frac{c}{B}$ are recoded and shown in Figure 6. As this figure shows in fixe ratios of $\frac{c}{B}$ by increasing in $\frac{a}{W}$ the frequencies ratios are decrease. It means that by growing the crack length in this direction the stiffness matrix of the beam was decrease and the beam became flexible. This decrease has higher intensity when $\frac{c}{B}$ become bigger. But according to Figure 6 some exception results was recorded.

By comparing the Figures 5 and 6, our results show that the intensity of growing crack in $\frac{a}{W}$direction is higher than the other two.Sothis directionaffected the beam's dynamic behavior with higher intensity than the other two.

5. FIGURES AND TABLES

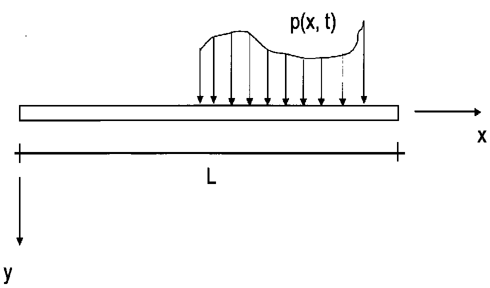

Figure 1. A beam of length L subjected to an external load$P(x, t)$.

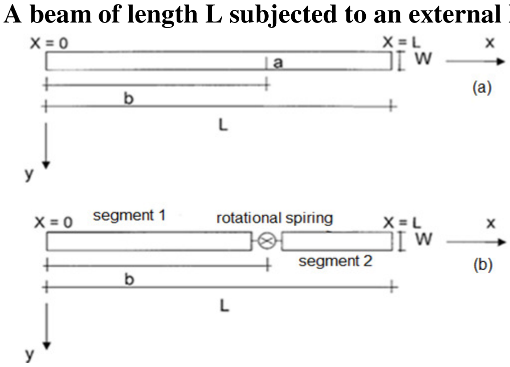

Figure 2. Cracked beam: (a) dimensions and position of the crack; (b) model of the cracked beam.[27]

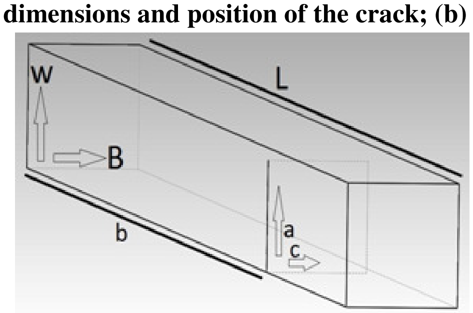

Figure 3. Cracked Cantilever Beam Model

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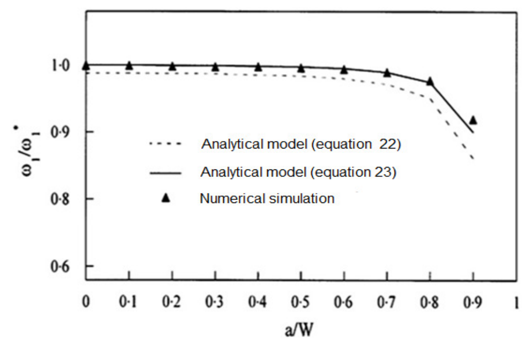

Figure 4. Variation of fundamental frequency with crack ratio of a cantilever beam

$$\left(\frac{c}{B} = 1, \quad \frac{b}{L} = 0.75\right)$$

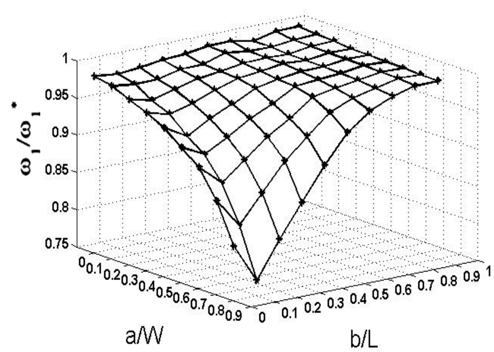

Figure 5. Variation of fundamental frequency with different $\frac{a}{W}$ and $\frac{b}{L}$ values of a cantilever beam ($\frac{c}{B} = 1$)

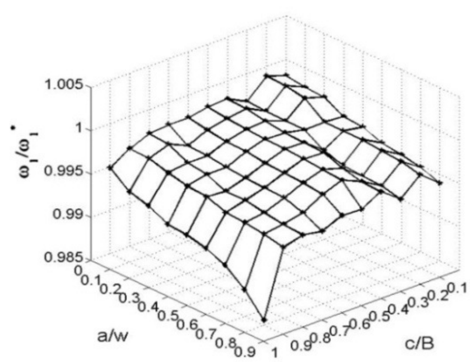

Figure 6. Variation of fundamental frequency with different $\frac{a}{W}$ and $\frac{c}{B}$ values of a Cantilever beam

($\frac{b}{L} = 0.75$)

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6. CONCLUSIONS

Crack detection and monitoring techniques run the full gamut from off-line visual techniques to a variety of non-destructive evaluation methods. In recent years, there have been a number of real-time crack detection technologies which are at various stages of implementation maturity. Cracks in a structure reduce its natural frequencies because it becomes more flexible. So, analysis and detecting of mechanical defects by monitoring of vibration behavior can be a good way for detecting cracks with respect to their size and location.

In this paper monitoring of cracks based on the first natural frequency (fundamental frequency) of cracked beams has been studied by authors. For this purpose, fundamental frequency of Euler–Bernoulli beam in bending vibration is obtained in two ways, analytical and numerical approaches. ANSYS workbench has been used regarding the numerical solution. The results of ANSYS workbench have been validated through comparison of them with the results of analytical solution. There is a good agreement between the analytical and numerical approaches. In continue, a few applied examples were simulated by FEM, regarding the location and size of the crack and the resulting fundamental frequency. In this regard, the data structure of 3D surfaces, as shown in Figure 5 and Figure 6, indicates the fundamental frequency of the cracked cantilever beam depending on the position and dimensions of the crack. The obtained results provide an insight into how fundamental frequency of the cracked structures will help in crack detection and monitoring.

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