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THE PRODUCT SPAN OF A FINITE SUBSET OF A COMPLETELY BOUNDED ARTEX SPACE OVER A BI-MONOID

ABSTRACT

The product of subsets of an Artex space over a bi-monoid is defined. Product Combination of elements of a Completely Bounded Artex Space over a bi-monoid is defined. Product span of a finite subset of a completely bounded Artex space over a bi-monoid is defined. A proposition is found and proved. Example is provided. The product of subsets of an Artex space over a bi-monoid is defined. Product Combination of elements of a Completely Bounded Artex Space over a bi-monoid is defined. Product span of a finite subset of a completely bounded Artex space over a bi-monoid is defined. A proposition is found and proved. Example is provided.

Keywords: Bi-monoids, Artex Spaces over bi-monoids, Completely Bounded Artex Spaces over bi-monoids,Product Combination,ProductSpan.

I .INTRODUCTION

There are many areas such as engineering and science to which Boolean algebra is applied. George Boole in 1854 introduced an algebraic system called Boolean Algebra. A more general algebraic system is the lattice. A Boolean Algebra is then introduced as a special lattice. Lattices and Boolean algebra have important applications in the theory and design of computers. As the theory of Artex Spaces over bi-monoids is developed from Lattice theory and linear algebra, theory of Artex Spaces over bi-monoids will be more useful. The algebraic system Bi-semi-group is more general to the existing algebraic system ring or an associative ring. Complete Artex Spaces over bi-monoids , Lower Bounded Artex Spaces over bi-monoids, Upper Bounded Artex Spaces over bi-monoids, Bounded Artex Spaces over bi-monoids and Completely Bounded Artex Spaces over a bi-monoids are very interesting and will be very useful. Product Combination of elements of a Completely Bounded Artex Space over a bi-monoid and Product span of a finite subset of a completely bounded Artex space over a bi-monoid will be useful to form many SubArtex spaces.

II.PRELIMINARIES

2.1Lattice : A lattice is a partially ordered set ($L,\leq$) in which every pair of elements $a,b \in L$ has a greatest lower bound and a least upper bound.

The greatest lower bound of a and b is denoted by $a\Lambda b$ and the least upper bound of a and b is denoted by $aV b$

2.2Lattice as an Algebraic System : A lattice is an algebraic system (L,Λ,V) with two binary operations Λ and V on L which are both commutative, associative and satisfy the absorption laws namely $a\Lambda(aVb) = a$ and $aV(a\Lambda b) = a$, for all $a,b\in L$

The operations Λ and V are called cap and cup respectively, or sometimes meet and join respectively.

2.3Properties : We have the following properties in a lattice (L , Λ,V)

1.$a \Lambda a = a$	1'.$a V a = a$	(Idempotent Law)
2.$a \Lambda b = b \Lambda a$	2'.$a V b = b V a$	(Commutative Law)
3.$(a \Lambda b)\Lambda c = a \Lambda(b \Lambda c)$	3'.$(a V b) V c = a V(b V c)$	(Associative Law)
4.$a \Lambda (a V b) = a$	4'.$a V (a \Lambda b) = a$, for all $a,b,c\in L$	(Absorption Law)

2.4 Complete Lattice : A lattice is called a complete lattice if each of its nonempty subsets has a least upper bound and a greatest lower bound.

Every finite lattice is a complete lattice and every complete lattice must have a least element and a greatest element.

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The least and the greatest elements, if they exist, are called the bounds or units of the lattice and are denoted by 0 and 1 respectively.

2.5 Bounded Lattice : A lattice which has both elements 0 and 1 is called a bounded lattice. A bounded lattice is denoted by (L, Λ,V,0,1)

The bounds 0 and 1 of a lattice (L, Λ,V) satisfy the following identities.

For any a∊L, a V 0 = a a Λ 1 = a a V 1 = 1 a Λ 0 = 0

2.5.1 Example : For any set S, the lattice (P(S), ⊆) is a bounded lattice.Here for each A,B ∊ P(S), the least upper bound of A and B is A∪B and the greatest lower bound of A and B is A∩B. The bounds in this lattice are φ, the empty set and S, the universal set.

2.6Complemented Lattice : Let (L, Λ,V,0,1) be a bounded lattice. An element a'∊L is called a complement of an element a∊L if a Λ a' = 0, a V a' = 1. A bounded lattice (L, Λ,V,0,1) is said to be a complemented lattice if every element of L has at least one complement. A complemented lattice is denoted by (L, Λ,V, ' ,0,1).

2.6.1Example : For any set S, the lattice (P(S), ⊆) is a Complemented lattice.

For each A,B ∊ P(S), the least upper bound of A and B is A∪B and the greatest lower bound of A and B is A∩B.

The bounds in this lattice are φ, the empty set and S, the universal set.

Here for any A ∊ P(S), the complement of A in P(S) is S-A

2.7 Doubly Closed Space: A non-empty set D together with two binary operations denoted by + and . is called a Doubly Closed Space if (i) a.(b+c) = a.b + a.c and (ii) (a+b).c = a.c + b.c, for all a,b,c ∊ D

A Doubly closed space is denoted by (D, + , .)

Note 1: The axioms (i) a.(b+c) = a.b + a.c and (ii) (a+b).c = a.c + b.c ,for all a,b,c ∊ D are called the distributive properties of the Doubly Closed Space.

Note 2: The operations + and **.** need not be the usual addition and usual multiplication respectively.

2.7.1Example : Let N be the set of all natural numbers.

Then (N, + , .), where + is the usual addition and . is the usual multiplication, is a Doubly closed space.

Similarly (Z, + , .), (Q, + , .), (R, + , .) and (C, + , .) are all Doubly closed spaces.

2.7.2Example : (Z , + , –), where + is the usual addition and – is the usual subtraction , is not a Doubly closed space.

Even though + and – are binary operations in Z, (Z , + , –) is not a Doubly closed space because of the distributive properties of the Doubly Closed Space.

Take a = 15, b = 7, c = 4

Then a – (b + c) = 15 – (7 + 4)
= 15 – 11
= 4

But (a – b) + (a – c) = (15 – 7) + (15 – 4)
= 8 + 11
= 19

Therefore, a – (b + c) ≠ (a – b) + (a – c)

Therefore, (Z , + , –) is not a Doubly closed space.

2.8Bi-semi-group : A Doubly closed space (S , + , .) is called a Bi-semi-group if + and . are associative in D.

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2.8.1Example : (N, + , .), (Z, + , .), (Q, + , .), (R, + , .), and (C, + , .), where + is the usual addition and **.** is the usual multiplication, are all Bi-semi-groups.

2.9Bi-monoid : A Bi-semi-group (M , + , .) is called a Bi-monoid if there exist elements denoted by 0 and 1 in S such that a+0=a=0+a, for all a∊M and a.1=a=1.a, for all a∊M .

The element 0 is called the identity element of M with respect to the binary operation + and the element 1 is called the identity element of M with respect to the binary operation**.**.

2.9.1Example : Let W = {0,1,2,3,…}.Then (W, + , .), where + is the usual addition and . is the usual multiplication, is a Bi-monoid.

2.9.2Example : Let Q'= Q^+∪{0}, where Q^+ is the set of all positive rational numbers. Then (Q', + , .) is a bi-monoid.

2.9.3Example : R'= R^+∪{0}, where R^+ is the set of all positive real numbers. Then (R', + , .) is a bi-monoid.

2.9.10Example : (Z, + , .), (Q, + , .), (R, + , .), and (C, + , .), where + is the usual addition and **.** is the usual multiplication, are all Bi-monoids.

2.10 Artex Space Over a Bi-monoid : Let (M , + , .) be a bi-monoid with the identity elements 0 and 1 with respect to + and **.** respectively. A non-empty set A together with two binary operations ^ and v is said to be an Artex Space Over the Bi-monoid (M , + , .) if

1.(A, Λ , V) is a lattice and

2.for each m∊M , m≠0, and a∊A, there exists an element ma ∊ A satisfying the following conditions :

(i) m(a Λ b) = ma Λ mb

(ii) m(a V b) = ma V mb

(iii) ma Λ na ≤ (m +n)a and ma V na ≤ (m + n)a

(iv) (mn)a = m(na), for all m,n∊M, m≠0, n≠0, and a,b∊A

(v) 1.a = a, for all a ∊ A.

Here, ≤ is the partial order relation corresponding to the lattice (A, Λ , V) .The multiplication ma is called a **bi-monoid multiplication with an artex element** or simply bi-monoid multiplication in A.

2.10.1 Example : Let W = {0,1,2,3,…}.

Then (W , + , .) is a bi-monoid , where + and **.** are the usual addition and multiplication respectively.

Let Z be the set of all integers

Then (Z, ≤) is a lattice in which Λ and V are defined by a Λ b = minimum of {a,b} and a V b = maximum of {a,b}, for all a,b∊Z.

Clearly for each m∊W,m≠0, and for each a∊Z, ma∊Z.

Also

(i) m(a Λ b) = ma Λ mb

(ii) m(a V b) = ma V mb

(iii) ma Λ na ≤ (m +n)a and ma V na ≤ (m + n)a

(iv) (mn)a = m(na)

(v) 1.a = a , for all m,n∊W, m ≠ 0, n ≠ 0 and a,b∊Z

Therefore, Z is an Artex Space Over the Bi-monoid (W , + , .)

Properties 2.11.1 : We have the following properties in a lattice (L , Λ,V)

1.a Λ a = a	1'.a V a = a
2.a Λ b = b Λ a	2'.a V b = b V a
3.(a Λ b)Λ c = a Λ(b Λ c)	3'.(a V b) V c = a V(b V c)
4.a Λ (a V b) = a	4'.a V (a Λ b) = a, for all a,b,c ∊ L

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Therefore, we have the following properties in an Artex Space A over a bi-monoid M.

(i)	m(a Λ a) = ma	(i)'.m(a V a) = ma
(ii)	(m(a Λ b) =m(b Λ a)	(ii)'.m(a V b) = m(b V a)
(iii)	m((a Λ b)Λ c)=m(a Λ(b Λ c))	(iii)'.m((a V b) V c) = m(a V(b V c))
(iv)	m(a Λ (a V b)) = ma	(iv)'.m(a V (a Λ b)) = ma,

for all $m \epsilon M$, $m \neq 0$ and $a,b,c \epsilon A$

2.12SubArtex Space : Let (A, Λ ,V) be an Artex space over a bi-monoid. (M , + , .). Let S be a nonempty subset of A. Then S is said to be a SubArtex Space of A if (S, Λ ,V) itself is an Artex Space over M.

2.12.1 Example : As defined in Example 3.2.1, Z is an Artex Space over W = {0,1,2,3,…..} and W is a subset of Z. Also W itself is an Artex space over W under the operations defined in Z. Therefore, W is a SubArtex space of Z.

2.13Complete Artex Space over a bi-monoid : An Artex space A over a bi- monoid M is said to be a Complete Artex Space over M if as a lattice, A is a complete lattice, that is each nonempty subset of A has a least upper bound and a greatest lower bound.

2.13.1Remark : Every Complete Artex space must have a least element and a greatest element.

The least and the greatest elements, if they exist, are called the bounds or units of the Artex space and are denoted by 0 and 1 respectively.

2.14Lower Bounded Artex Space over a bi-monoid : An Artex space A over a bi-monoid M is said to be a Lower Bounded Artex Space over M if as a lattice, A has the least element 0.

2.14.1Example : Let A be the set of all constant sequences (x_n) in $[0,\infty)$

Let W = {0,1,2,3,…}.

Define $\leq$ ', an order relation, on A by for (x_n), (y_n) in A, $(x_n) \leq' (y_n)$ means $x_n \leq y_n$, for each n

where $\leq$ is the usual relation " less than or equal to "

Therefore, A is an Artex space over W.

The sequence (0_n), where 0_n is 0 for all n, is a constant sequence belonging to A

Also $(0_n) \leq' (x_n)$, for all the sequences (x_n) belonging to in A

Therefore, (0_n) is the least element of A.

That is, the sequence 0,0,0,…… is the least element of A

Hence A is a Lower Bounded Artex space over W.

2.15Upper Bounded Artex Space over a bi-monoid : An Artex space A over a bi-monoid M is said to be an Upper Bounded Artex Space over M if as a lattice, A has the greatest element 1.

2.15.1Example : Let A be the set of all constant sequences (x_n) in $(-\infty,0]$ and let W = {0,1,2,3,…}.

Define $\leq$' ,an order relation, on A by for (x_n), (y_n) in A, $(x_n) \leq' (y_n)$ means $x_n \leq y_n$, for n = 1,2,3,…, where $\leq$ is the usual relation " less than or equal to "

A is an Artex space over W.

Now, the sequence (1_n), where 1_n is 0, for all n, is a constant sequence belonging to A

Also $(x_n) \leq' (1_n)$, for all the sequences (x_n) in A

Therefore, (1_n) is the greatest element of A.

That is, the sequence 0,0,0,….. is the greatest element of A

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Hence A is an Upper Bounded Artex Space over W.

2.16Bounded Artex Space over a bi-monoid : An Artex space A over a bi-monoid M is said to be a Bounded Artex Space over M if A is both a Lower bounded Artex Space over M and an Upper bounded Artex Space over M.

2.16.1Completely Bounded Artex Space over a bi-monoid: A Bounded Artex Space A over a bi-monoid M is said to be a Completely Bounded Artex Space over M if (i) 0.a = 0, for all a ϵ A (ii) m.0 = 0, for all m ϵ M.

2.16.2Note : While the least and the greatest elements of the Complemented Artex Space is denoted by 0 and 1, the identity elements of the bi-monoid (M , + , .) with respect to addition and multiplication are, if no confusion arises, also denoted by 0 and 1 respectively.

III. THE PRODUCT OF SUBSETS OF AN ARTEX SPACE OVER A BI-MONOID

3.1THE PRODUCT OF SUBSETS OF AN ARTEX SPACE OVER A BI-MONOID : Let (A, Λ ,V) be an Artex Space over a bi-monoid (M , + , .). Let S and T be subsets of the Artex Space A. Then the product of S and T denoted by S Λ T is defined by S Λ T = { s Λ t / s ϵ S and t ϵ T }

IV. THE PRODUCT SPAN OF A SUBSET OF A COMPLETELY BOUNDED ARTEX SPACE OVER A BI-MONOID

4.1Product Combination : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .). Let $a_1, a_2, a_3, \ldots\ldots\ a_n$ ϵ A. Then any element of the form $m_1a_1 \wedge m_2a_2 \wedge m_3a_3 \wedge \ldots\ldots \wedge m_na_n$, where m_i ϵ M, is called a Product Combination or Meet Combination of $a_1, a_2, a_3, \ldots\ldots\ a_n$ over the Artex Space A.

4.2The Product Span of a subset of a Completely Bounded Artex Space over a Bi-monoid : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .) and W be a nonempty finite subset of A. Then the Product Span of W or Meet Span of W denoted by P[W] is defined to be the set of all product combinations of elements of W. That is, if W = { $a_1, a_2, a_3, \ldots\ldots\ a_n$ }, then P[W] = { $m_1a_1 \wedge m_2a_2 \wedge m_3a_3 \wedge \ldots\ldots \wedge m_na_n$ / m_i ϵ M}.

PROPOSITION : 4.3.1 : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .). Let W and V be any two nonempty finite subsets of A. Then P[W∪V] = P[W] Λ P[V] .

Proof : Let (A, Λ ,V) be a Completely Bounded Artex Space over a bi-monoid (M , + , .)

Let W and V be any two nonempty finite subsets of A such that W = { $a_1, a_2, a_3, \ldots\ldots\ a_n$ } and V = { $b_1, b_2, b_3, \ldots\ldots\ b_k$ }

Let x ϵ P[W∪V]

Since Λ is commutative and associative, we may arrange x as

$x = m_1a_1 \wedge m_2a_2 \wedge m_3a_3 \wedge \ldots\ldots \wedge m_na_n \wedge m_{n+1}b_1 \wedge m_{n+2}b_2 \wedge \ldots\ldots \wedge m_{n+k}b_k$, where m_i ϵ M

Let $w = m_1a_1 \wedge m_2a_2 \wedge m_3a_3 \wedge \ldots\ldots \wedge m_na_n$ and let $v = m_{n+1}b_1 \wedge m_{n+2}b_2 \wedge \ldots\ldots \wedge m_{n+k}b_k$

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Then $x = w \Lambda v$

Clearly $w = m_1a_1 \Lambda m_2a_2 \Lambda m_3a_3 \Lambda \ldots\ldots \Lambda m_na_n \in P[W]$ and

$v = m_{n+1}b_1 \Lambda m_{n+2}b_2 \Lambda \ldots\ldots \Lambda m_{n+k}b_k \in P[V]$

Therefore, $x = w \Lambda v \in P[W] \Lambda P[V]$

Therefore, $P[W \cup V] \subseteq P[W] \Lambda P[V]$ ------------------ (i)

Conversely, let $x \in P[W] \Lambda P[V]$

Then $x = w \Lambda v$, where $w \in P[W]$ and $v \in P[V]$

Therefore, $w = m_1a_1 \Lambda m_2a_2 \Lambda m_3a_3 \Lambda \ldots\ldots \Lambda m_na_n$, $m_i \in M$ and

$v = m_{n+1}b_1 \Lambda m_{n+2}b_2 \Lambda \ldots\ldots \Lambda m_{n+k}b_k$, $m_i \in M$

Then $x = w \Lambda v$

$x = m_1a_1 \Lambda m_2a_2 \Lambda m_3a_3 \Lambda \ldots\ldots \Lambda m_na_n \Lambda m_{n+1}b_1 \Lambda m_{n+2}b_2 \Lambda \ldots\ldots \Lambda m_{n+k}b_k$, where $m_i \in M$

Therefore, $x \in P[W \cup V]$

Therefore, $P[W] \Lambda P[V] \subseteq P[W \cup V]$ ---------------------- (ii)

From (i) and (ii) we have $P[W \cup V] = P[W] \Lambda P[V]$.

4.4Example : Let $R' = R^+ \cup \{0\}$, where R^+ is the set of all positive real numbers and let $W = \{0,1,2,3,\ldots..\}$
$(R', \leq)$ is a lattice in which Λ and V are defined by $a \Lambda b =$ mini $\{a,b\}$ and $a V b =$ maxi $\{a,b\}$, for all $a,b \in R'$.
Here ma is the usual multiplication of a by m.
Clearly for each $m \in W, m \neq 0$, and for each $a \in R'$, $ma \in R'$.
Also,
(i) $m(a \Lambda b) = ma \Lambda mb$
(ii) $m(a V b) = ma V mb$
(iii) $ma \Lambda na \leq (m+n)a$ and $ma V na \leq (m+n)a$
(iv) $(mn)a = m(na)$, for all $m,n \in W$, $m \neq 0$, $n \neq 0$, and $a,b \in R'$
(v) $1.a = a$, for all $a \in R'$
Therefore, R' is an Artex Space Over the bi-monoid (W , + , .)

Generally, if Λ_1, Λ_2, and Λ_3 are the cap operations of A , B and C respectively and if V_1, V_2, and V_3 are the cup operations of A , B and C respectively, then the cap of $A \times B \times C$ denoted by Λ and the cup of $A \times B \times C$ denoted by V are defined

$x \Lambda y = (a_1,b_1,c_1) \Lambda (a_2,b_2,c_2) = (a_1 \Lambda_1 a_2 \Lambda_1 a_3 , b_1 \Lambda_2 b_2 \Lambda_2 b_3, c_1 \Lambda_3 c_2 \Lambda_3 c_3)$ and

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$x \vee y = (a_1,b_1,c_1) \vee (a_2,b_2,c_2) = (a_1 \vee_1 a_2 \vee_1 a_3, b_1 \vee_2 b_2 \vee_2 b_3, c_1 \vee_3 c_2 \vee_3 c_3)$

Here, Λ_1, Λ_2, and Λ_3 denote the same meaning minimum of two elements in R' and V_1, V_2, and V_3 denote the same meaning maximum of two elements in R'

Therefore, $R'^3 = R' \times R' \times R'$ is an Artex Space over W, where cap and cup operations are denoted by Λ and V respectively.

Let H = { (1,0,0) } and let T = { (0,1,0) }

Now P[H] = { (m,0,0) / m∈R' } and P[T] = { (0,n,0) / n∈R' }

P[H] Λ P[T] = { (m,0,0) / m∈R' } V { (0,n,0) / n∈R' }

= $\{(m \Lambda_1 0, 0 \Lambda_2 n, 0 \Lambda_3 0)\}$

= {(0,0,0)} (since $m \Lambda_1 0$ = mini.{m,0} = 0, $0 \Lambda_2 n$= mini.{0,n}=0 and $0 \Lambda_3 0$= mini.{0,0}=0)

P[H] Λ P[T] = {(0,0,0)} ----------- (i)

Now H ∪ T = { (1,0,0), (0,1,0) }

Let m,n ∈ M, m≠0,n≠0

Then m(1,0,0) Λ n(0,1,0) = (m,0,0) Λ (0,n,0)

= $(m \Lambda_1 0, 0 \Lambda_2 n, \Lambda_3 0)$

=(0,0,0) (since $m \Lambda_1 0$ = mini.{m,0} =0, $0 \Lambda_2 n$= mini.{0,n}=0 and $0 \Lambda_3 0$= mini.{0,0}=0)

Therefore, P[H∪T] = {(0,0,0)} ----------- (ii)

From equations (i) and (ii) we have P[H∪T] = P[H] Λ P[T]

V. CONCLUSION

From ArtexSpace to this Product Span we have come across so many miles. Product Combination of elements of an Artex Space over a bi-monoid , Product Span of a finite subset of a completely bounded artex space over a bi-monoid will create a new dimension in the theory of Artex spaces over bi-monoids. Interested researcher can do their research in product span.

References:

1. K.Muthukumaran and M.Kamaraj, "Artex Spaces Over Bi-Monoids", "Research Journal Of Pure Algebra", 2(5), May 2012, Pages 135-140.
2. K.Muthukumaran and M.Kamaraj, "Subartex Spaces Of An Artex Space Over A Bi-Monoid" ,"Mathematical Theory And Modeling"(With Ic Impact Factor 5.53), An Usa Journal Of "International Institute For Science, Technology And Education", Vol.2, No.7, 2012, Pages 39 – 48.
3. K.Muthukumaran and M.Kamaraj, "Bounded Artex Spaces Over Bi-Monoids And Artex Space Homomorphisms", "Research Journal Of Pure Algebra", 2(7), July 2012, Pages 206 – 216.
4. K.Muthukumaran and M.Kamaraj, "Some Special Artex Spaces Over Bi-Monoids", "Mathematical Theory And Modeling"(With Ic Impact Factor 5.53), An Usa Journal Of "International Institute For Science, Technology And Education", Vol.2, No.7, 2012, Pages 62 – 73.
5. K.Muthukumaran and M.Kamaraj, "Boolean Artex Spaces Over Bi-Monoids", Mathematical Theory And Modeling" (With Ic Impact Factor 5.53), An Usa Journal Of "International Institute For Science, Technology And Education", Vol.2, No.7, 2012, Pages 74 – 85.
6. K.Muthukumaran and M.Kamaraj, "Cap-Cosets And Cup-Cosets Of A Subset S In An Artex Space A Over A Bi-Monoid M", "IOSR Journal Of Mathematics", A Journal Of "International Organization of Scientific Research", Volume 2, Issue 1(July – August 2012) Pages 17 - 22.

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ISSN-2319-2119

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7. K.Muthukumaran and M.Kamaraj, "M-Artex Spaces Over Monoids And Cap-Quotient And Cup-Quotient M-Artex Spaces", "IOSR Journal Of Mathematics", A Journal Of "International Organization of Scientific Research" Volume 2, Issue 4(Sep –oc t 2012) Pages 22 - 28.
8. John T.Moore, The University of Florida /The University of Western Ontario, Elements of Abstract Algebra, Second Edition, The Macmillan Company, Collier-Macmillan Limited, London,1967.
9. J.P.Tremblay and R.Manohar, Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw-Hill Publishing Company Limited, New Delhi, 1997.
10. Garrett Birkhoff & Thomas C.Bartee, Modern Applied Algebra, CBS Publishers & Distributors,1987.
11. J.Eldon Whitesitt, Boolean Algebra And Its Applications, Addison-Wesley Publishing Company, Inc.,U.S.A., 1961.
12. I.N.HERSTEIN, Topics in Algebra, Wiley Eastern Limited.

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